Particle physics with the computer

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1   Introduction

As particle physics was evolving in the middle of the last century and more and more experimental data were collected, data analysis with the help of computers became important. Nowadays it is impossible to imagine a high-energy experiment, where each recorded event is analyzed separately by hand, like it was done in the early times of particle physics. There are several reasons for this. First of all, physicists are usually looking for rare processes employing beams of high intensity and high rates of data acquisition (up to 50 million events per second in modern experiments). Combined with typical beam times of several weeks or even months one can easily end up with a recorded event statistics of several billion events. Moreover, data delivered by contemporary detectors are complex and have to be processed in several steps.
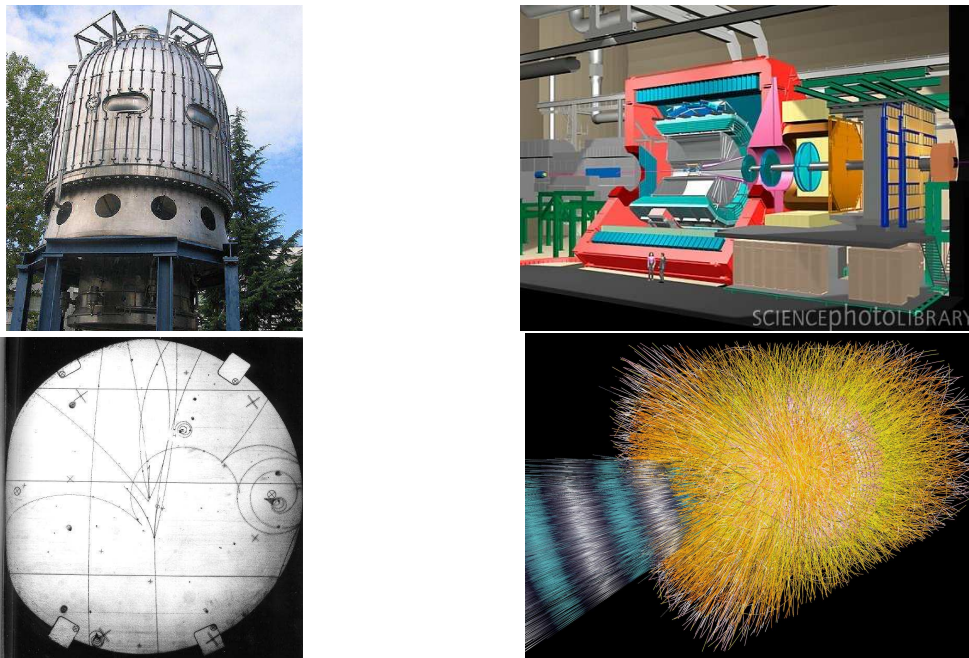
Figure 1: Left row: The Big European bubble chamber (BEBC) at CERN with a height of 4m. It was used with a filling of liquid hydrogen and operated in 1973 - 1984. Lower Figure: A typical bubble chamber picture showing a few particle tracks. Right row: The ALICE detector at CERN (2010) with a height of 16 m and a length of 26 m. Lower Figure: A simulated event in the ALICE detector of a lead lead collision.

With specialized software, the recorded digital signals are translated into interaction

points of particles with the detector (so-called hits). All interaction points in one recorded event are combined and complex computing algorithms reconstruct out of some thousand points real particle tracks giving the physicist "high-level" information like momentum, time-of-flight, energy, etc. of the detected particle. From this point the work of data analysis starts:

<div align="center">

to obtain interesting results

from a huge amount of collected information

with help of elaborated algorithms

implemented in computer programs.

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With this practicum we want to give an insight into the field of Data Analysis in particle physics experiments.

2 Basics of the analyzed experiment

The data which you will analyze within this lab were collected with the **H**igh **A**cceptance **D**i-**E**lectron **S**pectrometer (**HADES**) see fig 2. It is a fixed target
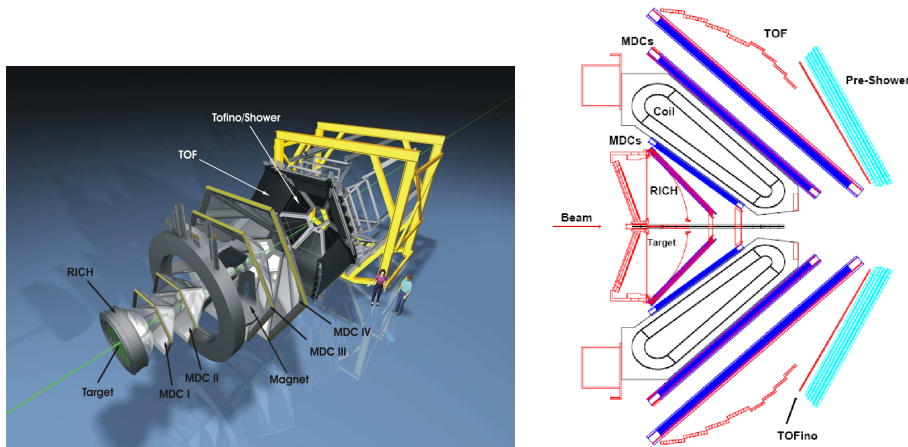

Figure 2: On the left side an exploded view of the HADES spectrometer is shown. On the right side you see a side view of the spectrometer with its many sub-detectors.

experiment located at the GSI research center in Darmstadt and currently performs measurements at the SIS18 accelerator with typical beam energies of a few GeV per nucleon. As follows from its name, the detector has a high acceptance, covering 85% of the azimuthal angle and polar angles from 15 to 85 degrees. This is a large spectrometer consisting out of many sub-detectors. Here is a short overview:

- The target. For the analyzed experiment a liquid hydrogen (LH_2) target was used. It had a length of 5 cm and was cooled down to a temperature of 20K.

- The **R**ing **I**maging **C**herenkov detector (RICH) for lepton identification. This detector is filled with a gas in which only leptons create Cherenkov light when traversing this detector. In the energy regime where the spectrometer records events this detector is completely blind for hadrons.

- The 4 planes of multi wire drift chambers (MDC's). These drift chambers are employed for particle tracking and momentum reconstruction. The chambers are filled with a gas mixture and contain six layers of wire planes where avalanche signals caused by particles traversing the chambers can be registered.

- The Magnet. Between the MDC planes I, II and III, IV the superconducting magnet is located. It provides a toroidal magnetic field and is used for the momentum determination of the particles. By traversing the magnetic field each particle is bended according to its momentum. The change in flight direction can then be measured by the MDC chambers and the momentum of the particle can be reconstructed.

- The time-of-flight Detectors. These scintillating detectors are located at the very end of the spectrometer. Their quality is a very good time resolution. They are employed to measure the time which particles need to fly from the target, through the spectrometer, to the TOF detectors. Together with the measured trajectory length, the particle velocity can be reconstructed.

In summary the purpose of the Spectrometer is to reconstruct the four-momentum of a particle with the help of sub-detectors.

Let us remind how a four-vector looks like:

$$p = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} \tag{1}$$

The three-momentum components are directly measured by the spectrometer. To determine the energy of a particle, which can be also written like $E = \sqrt{p^2 + m^2}$, one needs to know additionally its mass. The mass of a particle is uniquely defined by its type: a proton, a pion, a kaon etc. Therefore, to obtain the full 4-vector one has to perform a so called particle identification.

3 Particle identification

3.1 dE/dx vs Momentum

If particles traverse material they loose energy. The energy loss can be hereby described analytically by the Bethe-Bloch formula.

$$-\frac{dE}{dx} = \frac{4\pi n z^2}{m_e c^2 \beta^2} \cdot \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_e c^2 \beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right] \tag{2}$$

with $\beta = v/c$, v the velocity of the particle, E the energy of the particle, x the distance traveled by the particle, c speed of light, $z \cdot e$ the particle charge, e the charge of the electron, m_e the rest mass of the electron, n the electron density of the traversed material and I the mean excitation potential of the material.

As the energy loss is independent of the mass of the traversing particle one obtains different energy loss curves for different particle species, when plotting the dE/dx vs the measured momentum of the particle.

In case of the HADES spectrometer the energy loss of particles can be measured in the MDC chambers and the TOF detectors. In fig. 3 you see the energy loss of the particles in the MDC chambers plotted against their momentum.
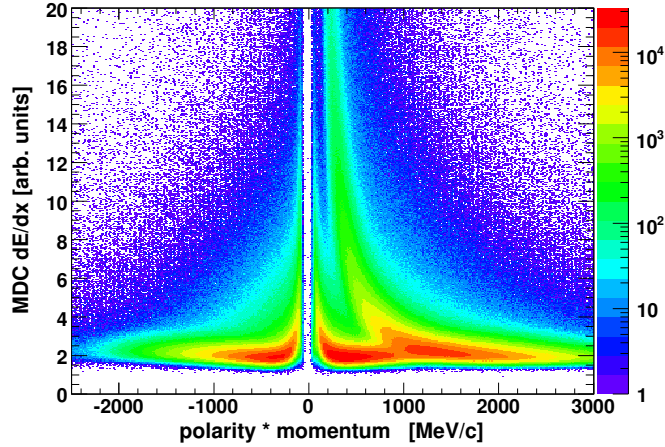

Figure 3: The energy loss (dE/dx) plotted against the particles momentum times their charge.

Your Task: Can you name the different particles you see on fig. 3?

3.2 Beta vs Momentum

Another way to identify particles is to measure not the energy loss but the time of flight of the particles. With help of the time of flight and the path length of the particle one can calculate the particles velocity (β). We can write the particles momentum as a function of the velocity as follows:

$$p = \frac{m \cdot v}{\sqrt{1 - \beta^2}}, \tag{3}$$

or

$$p/c = \frac{m \cdot \beta}{\sqrt{1 - \beta^2}}, \tag{4}$$

Then one can easily derive:

$$m = \frac{p}{c}\sqrt{\frac{1}{\beta^2} - 1} \tag{5}$$

The last formula shows that in order to identify the mass of a particle, it is enough to measure simultaneously its momentum and velocity.

4 Analyzing Events

In April 2007 the HADES experiment collected $1.2 \cdot 10^9$ events of proton-proton collisions with the proton-beam having a kinetic energy of 3.5 GeV. For the lab a subset of the collected statistics will be provided.

As a result of the collision new particles are created out of the energy the beam proton imposes on the target. For example:

$$p + p \rightarrow p + p \tag{6}$$
$$p + p \rightarrow p + p + \pi^0 \tag{7}$$
$$p + p \rightarrow \pi^0 + \pi^+ + \pi^+ \tag{8}$$
$$p + p \rightarrow p + K^+ + \Lambda \tag{9}$$
$$p + p \rightarrow p + n + e^+ \tag{10}$$
$$p + p \rightarrow p + p + \omega \tag{11}$$
$$p + p \rightarrow \Sigma^+ + K^0 + p \tag{12}$$
$$p + p \rightarrow \Sigma^- + K^+ + p + \pi^+ \tag{13}$$

Your Task: Which of these reactions are not possible and why? Think of conservation laws which have to be fulfilled.

Some of the particles appearing in reactions 6-13 are short-living particles which have lifetimes in the order of ns and decay lengths of few fm[1]. It is impossible to measure such particles directly. However, in order to reconstruct these particles, one can employ two methods which are based on the laws of energy and momentum conservation.

4.1 Invariant mass

Two variables, commonly used in high-energy physics are invariant and missing masses. The invariant mass of two particles is defined as the sum of the particles 4-vectors squared:

$$M_{Inv} = (P_1 + P_2)^2.$$

The invariant mass can be also expressed as:

$$M_{Inv} = m_1^2 + m_2^2 + 2\left(E_1 E_2 - |p_1| \cdot |p_2| \cdot \cos\theta\right).$$

---

[1] **fm** stands for femtometer (or, an obsolete name, fermi). It is a common unit of length in particle physics. 1 fm $= 10^{-15}$ m

Since the invariant mass is a square of the 4-vector, this variable—as its name suggests—is invariant under Lorenz transformations. Short-living particles can be reconstructed with this simple method.

Note, that the invariant mass can be constructed for any amount of particles:

$$M_{Inv}^n = (P_1 + P_2 + \ldots + P_n)^2.$$

This definition is used for example when studying 3- and 4-body decays.

Example:
The decay of $\Lambda \to p\pi^-$. The Λ has a lifetime of ≈ 0.26 ns. It is thus able to fly only ≈ 8 cm before decaying. In our Spectrometer we will therefore see no direct evidence for the Λ particle and it can be only reconstructed by its decay products.
The decay of $\eta \to \pi^+\pi^-\pi^0$. In this case one even needs to detect three particles and build the invariant mass of their Lorentz Vectors to obtain an η signal.

4.2   Missing mass

In case of elementary reactions (p + p, π + p, ...) it is also possible to reconstruct the event topology by means of the missing mass method. For this method one uses the Lorentz vectors of the incoming beam and the target. If one reconstructs in an event all but one particles in the final state one can identify the missing particle with the following method:

$$M_{missed} = (P_{Beam} + P_{Target} - P_1 - P_2 - \ldots - P_n)^2.$$

Thus a missing mass technique can be employed for the reconstruction of particles that were not detected.

Example:
Assuming we have measured the reaction

$$\pi^- + p \to \Lambda + K_S^0.$$

and the experiment allows to reconstruct the K^0 via its decay into a $\pi^+\pi^-$-pair. We can try to reconstruct the Λ also by its decay products or apply the missing mass technique. A 4-momentum conservation for this reaction reads as follows

$$P_{\pi^-} + P_p = P_\Lambda + P_{K_S^0}.$$

In this case, the missing mass of the reconstructed K^0 will be:

$$M_{miss}^{K^0} = (P_{\pi^-} + P_p - P_{\pi^-} - P_{\pi^+})^2 \quad \hat{=} \quad (P_\Lambda)^2 = M_\Lambda.$$

Your Task: Why in case of ion collisions, it is complicated, or even impossible to apply the missing mass technique?

5 Kinematics of elastic collisions

An elastic collision is a special type of reaction in which no new particles are produced and the colliding particles do not change their type. Examples are collisions of two billiard balls or proton-proton elastic scattering: $p + p \rightarrow p + p$. In case the physicist wants to identify elastic events he or she can exploit the special kinematics of the reaction.

Let us consider a collision of a beam particle P_B with a target particle P_T and two particles in the final state.

$$P_B + P_T \rightarrow P_1 + P_2 \tag{14}$$

Since only two particles in the final state are produced, there are 8 kinematic degrees of freedom represented by the components of the two four-vectors. Two components are fixed by the masses of the particles, three by momentum conservation and one by energy conservation. Therefore, only two degrees of freedom remain. This means in the case of an elastic collision one only needs to measure two observables and deduce all components of the four-vector.
Example:
We measure $\vec{p}_{1,x}$ and $\vec{p}_{1,y}$. Using momentum conservation we can write:

$$0 = \vec{p}_{1,x} + \vec{p}_{2,x} \tag{15}$$
$$0 = \vec{p}_{1,y} + \vec{p}_{2,y} \tag{16}$$

We also know that

$$\vec{p}_{B,z} = \vec{p}_{1,z} + \vec{p}_{2,z} \tag{17}$$

And (from energy conservation)

$$E_{B,Kin} + 2 \cdot m_p = \sqrt{\vec{p}_{1,x}^2 + \vec{p}_{1,y}^2 + \vec{p}_{1,z}^2 + m_p^2} + \sqrt{\vec{p}_{2,x}^2 + \vec{p}_{2,y}^2 + \vec{p}_{2,z}^2 + m_p^2} \tag{18}$$

As in an experiment $E_{B,Kin}$ and $\vec{p}_{B,z}$ are normally known, it is simple to derive the whole kinematics from measuring $\vec{p}_{1,x}$ and $\vec{p}_{1,y}$.

But we can also derive the elastic kinematics by measuring the deflection angles of the outgoing particles. A very natural frame for studying elastic events is the Center of Mass frame (CM, noted with *) in which target and beam vectors sum up to zero. (The system in which the target is at rest is called Target System,
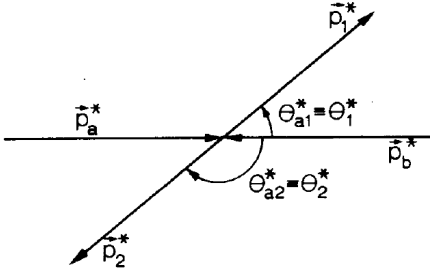
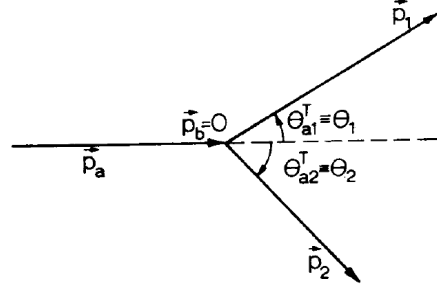Figure 4: Angles of an elastic collision in the CM-System

Figure 5: Angles of an elastic collision in the Target-System

TS or Laboratory System). Moreover one would consider here instead of Cartesian coordinates spherical coordinates.

$$r = \sqrt{p_x^2 + p_y^2 + p_z^2} \tag{19}$$

$$\varphi = \arctan\left(\frac{p_x}{p_y}\right) \tag{20}$$

$$\theta = \arccos\left(\frac{p_z}{|\vec{p}|}\right) \tag{21}$$

By energy and momentum conservation in the CM system r is fixed. The angles φ and θ are the two free parameters. The momentum conservation in this case reads as follows:

$$\theta_1^* = \pi - \theta_2^* \tag{22}$$
$$\varphi_1^* = \pi - \varphi_2^* \tag{23}$$

The big advantage of choosing this coordinate system is that one does not need to measure the particles momentum. However, in order to use these relations in practice, we would have to transform the measured four-vectors from the TS into the CM system. For this we would need to know the momentum components. Instead we transform the relations (22) and (23) to the TS frame. As the beam is (by the definition of the coordinate system) collinear with the z-axis one only has to perform a Lorentz boost in z-direction for a transformation from TS to CMS. This means that the transverse component ($\vec{p}_\perp$) of the vector does not change under this Lorentz transformation:

$$\vec{p}_\perp = |p_1| \cdot \sin\theta_1 = |p_1^*| \cdot \sin\theta_1^* \tag{24}$$

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The only reason why $|p^*|$ and θ^* obtain new values in the TS frame is the fact, that the momentum component parallel to the beam direction ($\vec{p}_{||}$) changes. This can be expressed as follows:

$$\vec{p}_{||} = |p_1| \cdot \cos\theta_1 = \gamma_{pp}|p_1^*| \cdot \cos\theta_1^* - \gamma_{pp}\beta_{pp}E_1^* \tag{25}$$

The transformation is determined by β_{pp} and γ_{pp} which are the velocity and the Lorentz factor of the initial p+p system. To obtain a transformation of the θ angle from the CM into the TS frame without a dependency on p_1 we divide (24) by (25) and obtain:

$$\tan\theta_1 = \frac{\sin\theta_1^*}{\gamma_{pp}(\cos\theta_1^* - \beta_{pp}\frac{E_1^*}{|p_1^*|})} \tag{26}$$

$$\tan\theta_1 = \frac{\sin\theta_1^*}{\gamma_{pp}(\cos\theta_1^* - \frac{\beta_{pp}}{\beta_1^*})} \tag{27}$$

If we use relation (22) that connects θ_1^* and θ_2^* we obtain:

$$\tan\theta_2 = \frac{\sin(\pi - \theta_1^*)}{\gamma_{pp}(\cos(\pi - \theta_1^*) - \frac{\beta_{pp}}{\beta_1^*})} \tag{28}$$

$$\tan\theta_2 = \frac{\sin\theta_1^*}{\gamma_{pp}(-\cos\theta_1^* - \frac{\beta_{pp}}{\beta_1^*})} \tag{29}$$

The value for $\frac{\beta_{pp}}{\beta_1^*}$ can be derived using the following relations:

$$E_1^* = \frac{m_{pp}^2 + m_1^2 - m_2^2}{2m_{pp}} \tag{30}$$

$$|\vec{p}_1^*| = \frac{[(m_{pp}^2 - (m_1 - m_2)^2)(m_{pp}^2 - (m_1 + m_2)^2)]^{1/2}}{2m_{pp}} \tag{31}$$

and

$$E_{pp} = \frac{m_{pp}^2 - m_B^2 + m_T^2}{2m_T} \tag{32}$$

$$|\vec{p}_{pp}| = \frac{[(m_T^2 - (m_{pp} - m_B)^2)(m_T^2 - (m_{pp} + m_B)^2)]^{1/2}}{2m_T} \tag{33}$$

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m_1 and m_2 are the masses of the two outgoing particles. m_B and $|\vec{p}_B|$ are the mass and momentum of the beam particle. m_T is the mass of the target particle. m_{pp} and E_{pp} are here the mass and the energy of the combined vector of the target and beam proton. As the relativistic velocity β is defined as $\frac{p}{E}$ we obtain $\frac{\beta_{pp}}{\beta_1^*} = 1$.

Your Task: Derive equations (30) and (32). Can you connect $\tan\theta_1$ and $\tan\theta_2$ without any dependence on angles? What is the value for γ for a p+p reaction with a beam kinetic energy of 3.5 GeV?

5.1   Selecting elastic collision events in an experiment

We have learned so far that two measured observables in a reaction are enough to determine the full kinematics of elastic events. So applying cuts on $\tan\theta_1$, $\tan\theta_2$ and φ_1, φ_2 would be sufficient to determine completely the four-vectors of both protons for elastic events. When the event was however, a three body production instead of a two body reaction we have 3 more degrees of freedom.

Note that in this case we need to measure 5 observables to explicitly determine all components of the four-vectors. In case we explicitly measure 3 particles with our detector there is no doubt that it was not an elastic scattering reaction but if we have one or more missing particles we could see if for example the p_x and p_y components of our two protons add up to zero. There are however cases where this still could add up to roughly zero but is not a sign of a two body reaction.

This means that asking for two constraints in our data analysis is a necessary constraint but not a sufficient one. An easy way to check if there was another particle produced is to investigate the missing mass of the two protons. We expect to see the mass of the third particle with help of this observable.

6 Reaction cross section

A value used in particle physics to characterize the probability of the particles interaction is a cross section. The reaction cross section is defined as

$$\sigma = \frac{R_{reac}}{\Phi},$$

where R_{reac} is a reaction rate for one target particle (number of reactions per unit time), Φ is a particle flux (amount of bombarding particles per unit area and per unit time, $\frac{1}{cm^2 \cdot s}$).

The unit of a cross section is defined as barn:

$$1 \ b = 10^{-28} \ m^2.$$

For particle reactions one normally uses milli- and microbarns, $10^{-3} \cdot b$ and $10^{-6} \cdot b$, respectively. Fig. 6 shows the energy dependence of the proton-proton cross section. Data on elastic cross section can be found in Appendix A.
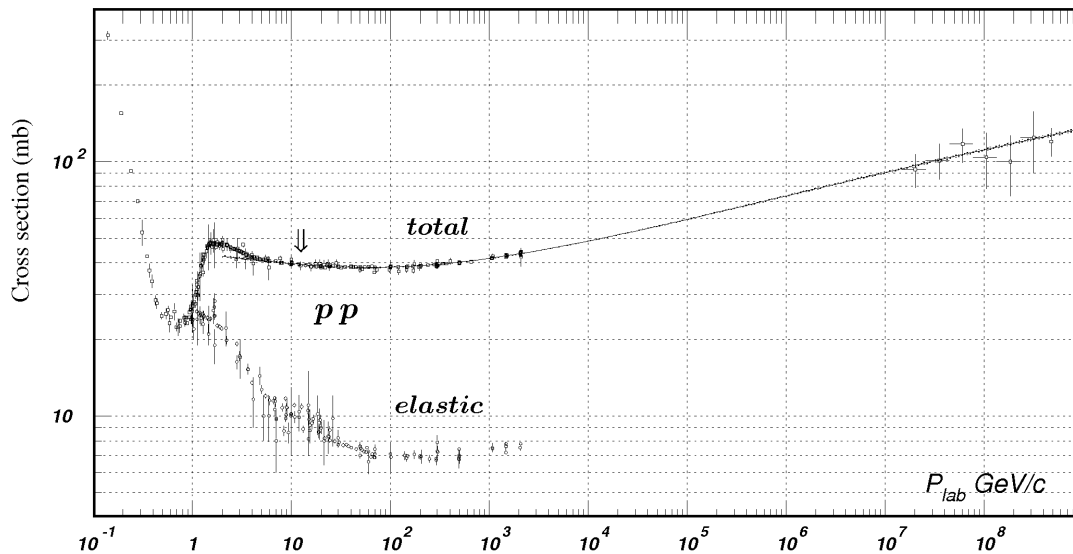

Figure 6: Total and elastic cross section of proton-proton collisions as a function of the proton momentum in the laboratory reference frame [PDG]

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7 Shell commands

In the course of the practicum, a minimal knowledge of Linux console (shell) commands is required.

Figure 7: Left: The symbol on your desktop to open the Terminal. Right: an open terminal where you can type your commands.

Here is a list of the most important commands. To execute a command, write it in the console and press enter.

Show the current directory. This is so to say the folder in which you work at the current moment:

```
pwd
```

Change the current directory to directory named "`dirname`" (with this command change between folders):

```
cd dirname
```

Show a list of the files in the current directory:

```
ls
```

To open C files you can use the text editor fte or any other editor you know and like. You have to type in this case:

```
fte program_name.cc
```

To compile the program (make an executable file out of the program code):

```
make clean build install
```

Run the (executable) program with name "`program_name`".

`./program_name`

A very comfortable command is the tab option. If you start to write a directory path or a macro name you can type the first letters and press the tab button on your keyboard. The complete name will then be auto-filled if this is the only possibility if however several possibilities exist how the name could be completed this option will print you all possibilities in your shell.

8 Basic C-operators

Comments
In C and C++ comments are marked with `//`. Any command written after this sign will be ignored.

Variable creation
To create a variable one need to specify its type (integer, float or other) and its name:

```
Int_t var1;       //integer type: 5, 100, -10
Float_t var2;     //float type: 3.4, 3.14156
```

Note, that in C and C++ lines of code (different commands) are always separated with a semicolon. A forgotten semicolon is a common mistake while programming.

Conditional statement
in C, a conditional statement has a following form:

```
if (condition) {
  perform actions
}
```

for example, we fill a histogram, only if the condition that $\sqrt{(p_x^2 + p_y^2)}$ is greater 25.5 is fulfilled

```
if ( sqrt(px*px+py*py) > 25.5) {
  h1->Fill(var1);
}
```

To build a logical condition there is a set of logical operators. Most of them have intuitive meaning:

C-operator	Meaning
a>b	a is greater than b
a<b	a is less than b
a==b	a is equal to b
a!=b	a is not equal to b
a&&b	logical AND
a||b	logical OR

Table 1: Logical operators in C

Arithmetic operators

C-operator	Meaning
+	add
−	subtract
*	multiply
/	divide
sqrt(a)	square root of a
pow(a,b)	a^b
fabs(a)	$|a|$

Table 2: Arithmetical operators in C

for-loop

C supports a number of loop types. For our purposes the so called **for-loop** is enough. In such a loop a variable (in this case `i`) is defined. Each loop iteration the variable is increased by a value of 1 (`i++`), until it reaches its maximum allowed valued (`i<100`).

```
for (Int_t i = 0; i < 100; i++) {
  h1->Fill(i);    //filling the histogram
  ... //any commands on this place
  ... //any commands on this place
}
```

In this case, any operators on the place of "..." will be repeated exactly 100 times (integer variable `i` will take values from 0 to 99).

Also very useful is the `cout` command.

`cout<<"Hello world"<<endl;`

will print you the word `Hello world` on your screen when executed. But you can also display variables in the code.

`cout<<"This is the value of beta : "<<beta1<<endl;`

will print you the actual value of beta1 on your screen. For example

`This is the value of beta : 0.72`

9 ROOT: a class packet for data analysis

ROOT is a specialized software package that was developed at CERN to serve the needs of the particle physics community, especially for the data processing of the experimental set-ups located at the LHC.

It is a C++ library that contains a number of useful classes. Here a basic knowledge about working with ROOT will be given.

9.1 Working with 4-vectors

A dedicated class `TLorentzVector` to work with 4-vectors.

Create a 4-vector object with the name "`particle_a`"

```
TLorentzVector particle_a;
```

Set the components of a 4-vector:

```
particle_a.SetPxPyPzE(px, py, pz, E);
```

One can for example access easily polar and azimuthal angles θ, ϕ (in radians) and rapidity y of the 4-vector:

```
Float_t theta = particle_a.Theta();
Float_t phi = particle_a.Phi();
Float_t y = particle_a.Rapidity();
```

A `TLorentzVector` class provides a convenient way to construct the sum of two and more 4-vectors and calculate invariant and missing masses:

```
//construct a 4-vector named pairAB
//out of two 4-vectors particleA and particleB
TLorentzVector pairAB;
pairAB = particleA + particleB;

//calculate invariant mass of the 4-vector pairAB
Float_t M_inv = pairAB.M();

//construct a missing 4-vector, given initial (beam+target) 4-vector
TLorentzVector missed;
```

```
missed = initial - pairAB;

//access the value of the missed mass
Float_t M_missed =  missed.M();
```

9.2   Working with histograms

A histogram is a convenient and common way to represent the statistical behavior of one or two variables. ROOT contains a set of classes to work with histograms. For our purposes two of them are enough: TH1F and TH2F.

To create a histogram named h1 with **500** bins in the region of an X-value from **0** to **5**:

```
TH1F *h1 = new TH1F("h1","Histogram title",500,0,5);
```

To add an entry to the histogram, just use the Fill() method. The entry in this case should be either a number:

```
h1->Fill(2.3);
```

or a variable (named, in this example, "theta_1"), containing a number:

```
h1->Fill(theta_1);
```

After the histogram is filled with values, one can draw it:

```
h1->Draw();
```

Let's now turn to 2-dimensional histograms. They are in particular helpful, when one would like to analyze a correlation between two variables, for example angle θ and momentum p of a particle.

To work with them is very similar to the 1-dimensional case, but now there are two axes:

To create a **two-dimensional** histogram named h2 with **500** bins in the region of X-values from **0** to **5** and **100** bins in the region of Y-values from **-10** to **10** :

```
TH2F *h2 = new TH2F("h2","Histogram title",500,0,5,100,-10,10);
```

To add an entry to the two-dimensional histogram, one need to provide two values (x,y)

```
h1->Fill(4.5, -7);
```

or

```
h1->Fill(theta_1, theta_2);
```

Drawing of a histogram is completely analogous to the 1-dimensional case:

```
h1->Draw();
```

Much more information about histogram usage can be found in Chapter 3 of the ROOT User's Guide.

10 Data for the analysis

The data for the analysis is prepared in form of an ntuple. An ntuple can be understood as a simple table that contains different numerical values. Each row in such a table corresponds to one event, and each column to one variable (see Table 3).

Event number	Variable 1	Variable 2	...
0	100.1	-200.5	...
1	437.7	289.3	...
...	...	...	...

Table 3: General logical structure of an ntuple

The ntuple prepared for the analysis was preselected to contain only events where two particles were detected in the spectrometer. It contains several observables. In each event the ntuple is filled with the extracted values:

Variable name	Meaning
charge1	sign of the particle charge ($+1$ or -1)
beta1	$\beta = v/c$, where c is the speed of light
px1	p_x, x-component of the particle momentum
py1	p_y, y-component of the particle momentum
pz1	p_z, z-component of the particle momentum
dedx1	particle energy loss in the detector (MDC chambers)

Table 4: Variables in the ntuple (for one particle) and their meaning

Since the prepared data sample always contains two reconstructed particles per event, the ntuple contains exactly the same set of variables for particle 2 (charge2, beta2, etc.).

Usually, a routine step in data reduction is to perform an iteration over the data (in our case to go through each entry in the ntuple), select an interesting data sub-sample, perform calculations when necessary, and obtain a result — typically a number, or series of numbers.

11 Tasks

For the completion of the practicum tasks use the provided template program. It already contains an iteration over all events. And some skeleton of basic code.

11.1 Creation of 4-vectors

The first task is to construct out of the observables contained in the ntuple two 4-vectors (with help of the `TLorentzVector` class) for particle 1 and particle 2.

11.2 Analysis of proton-proton elastic collisions

After the construction of two 4-vectors you have all the needed information to select events with proton-proton elastic collisions. The aim of this task is to develop a strategy to select elastic events and estimate their amount.

1. What kind of observables can you use to select pp-elastic events?

2. Make histograms (one or two-dimensional) of the observables you would like to constrain on.

3. Apply a constraint to one observable and see how the distribution of the other observables change.

4. Introduce step by step more cuts until you think your events are pure enough and document your analysis strategy with histograms. A general aim of a data analysis is to select the targeted reaction with as low background as possible. However, if the cut conditions applied are too strict you will, obviously, loose a lot of statistic. Try to find a balance between efficiency (amount of signal) and purity (amount of background events which remain after the cuts) of your data analysis.

5. Estimate the total amount of elastic proton-proton collisions after your final selection. For this select one final observable and plot this observable with selective constrains on other observables.

11.3 Analysis of a $p + p \rightarrow p + p + \pi^0$ reaction

The reconstruction of two protons in the final state is enough to select the reaction $p + p + \pi^0$.

1. Construct an observable to select events with π^0.

2. Plot the chosen observable and identify events that correspond to the $p+p+\pi^0$ reaction.

3. Is there a way to purify the data sample of events containing two protons and a π^0?

4. Estimate the number of $p + p + \pi^0$ events. In this case of the event selection the signal will be small as compared to the background underneath the signal. Take a prepared function to fit the signal together with the background and extract with help of the fit function $N_{p+p+\pi^0}$.

11.4 Estimation of a cross-section of the $p + p \rightarrow p + p + \pi^0$ reaction

As explained in section 6 of this description, a cross section of the reaction characterizes its probability. Let us label the number of observed elastic proton-proton collisions as N_{pp} and number of $p + p + \pi^0$ reactions as $N_{pp\pi^0}$. Then

$$\frac{N_{pp}}{N_{pp\pi^0}} = \frac{\sigma_{pp}}{\sigma_{pp\pi^0}},$$

where σ_{pp} and $\sigma_{pp\pi^0}$ are the cross sections of pp-elastic and $p + p + \pi^0$ reactions, respectively.

1. Extract the number of pp elastic events N_{pp} from 11.2 point 5.

2. Use the Appendix A to interpolate the elastic cross section at the beam momentum of the analyzed experiment.

3. Given a known cross-section of pp-elastic events and using the ratio between the number of pp-elastic events and $p + p + \pi^0$ production, calculate the cross section of the $p + p + \pi^0$ reaction.

4. Compare the obtained cross-section with the known value.

Here we made an important assumption, that the acceptance of the events of $p + p$ and $p + p + \pi^0$ reactions is the same. In general, the acceptance can depend very strongly on the reaction. In our case, this assumption is acceptable (why?). Are there other reasons why the two results are not direct comparable?

After completion of all the task a short report has to be written, summarizing the analysis strategy and the obtained results.

A   p + p elastic cross sections

Here is the measured data on proton-proton elastic cross section, taken from the PDG web page. The data is organized in the following columns: 1) measurement number, 2) beam momentum (GeV/c) in the laboratory frame, 3) cross section (mb), 4) statistical uncertainty of the measurement (mb).

NUMBER	PLAB(GEV/C)	SIG(MB)	ERR
01	2.22883	19.860	0.73000
02	2.80000	16.300	1.0000
03	2.80553	19.210	0.48000
04	3.00000	17.200	0.70000
05	3.03662	17.000	3.0000
06	3.65000	15.200	0.60000
07	3.67024	15.320	0.76000
08	4.00000	13.500	0.30000
09	4.15000	11.600	2.6000
10	4.80000	14.400	1.2000
11	5.00000	12.700	0.50000
12	5.25517	10.000	2.0000
13	5.52000	11.990	0.25000
14	5.96493	10.000	2.1000
15	6.00000	11.500	0.50000
16	6.60000	11.470	0.33000
17	6.79990	11.790	0.22000
18	6.80000	10.600	0.60000
19	6.92000	11.400	0.50000
20	7.02590	8.0000	2.0000
21	7.07634	9.7000	1.0000
22	7.07634	9.8000	0.90000
23	8.10000	10.800	0.40000
24	8.50000	8.7400	0.40000
25	8.80000	11.710	0.22000
26	8.80000	9.8000	0.30000
27	8.90000	10.100	0.50000
28	9.00000	10.840	0.32000