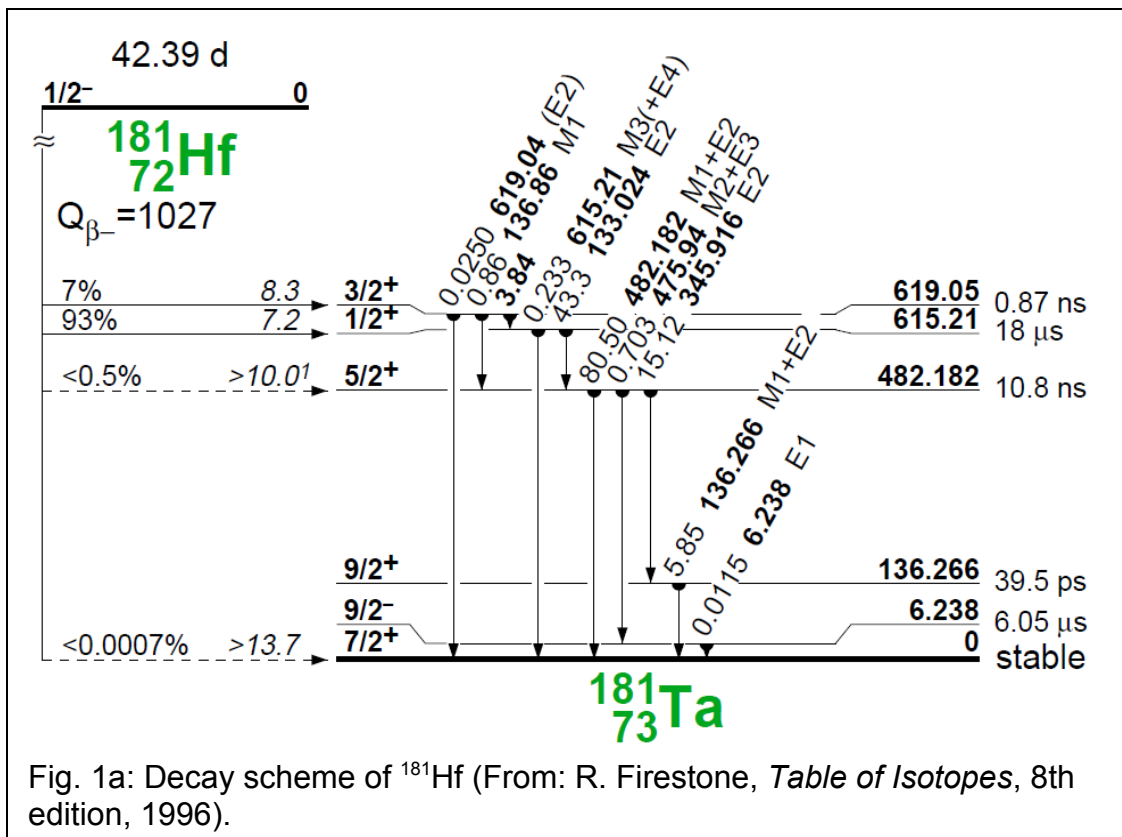


## Lifetime measurement of excited nuclear states by delayed $\gamma$ - $\gamma$ coincidences

(Remark: The experiment is called „Lifetime measurement“ but we are measuring a half-life of an excited state. Lifetime  $\tau$  and half-life  $t_{1/2}$  are linked via  $t_{1/2} = \tau \cdot \ln 2$ .)

In this experiment we measure the half-life of the  $5/2^+$  state in  $^{181}\text{Ta}$  at  $E_\gamma=482$  keV. The used experimental method is a delayed  $\gamma$ - $\gamma$  coincidence. The  $5/2^+$  state can be populated by the  $\beta^-$  decay of  $^{181}\text{Hf}$  ( $t_{1/2} = 42.39$  d) into  $^{181}\text{Ta}$ . The decay scheme is shown in Fig. 1a, Fig. 1b shows all of the  $\gamma$  transitions (in keV) and their respective intensities (in %).



$\gamma(^{181}\text{Ta})$  from  $^{181}\text{Hf}$  (42.39 d)  $\beta^-$  decay <for  $I_\gamma\%$  multiply by 0.8054>

3.90 10(?) ( $t_{1/2} = 5.627$ )  
 6.33 ( $t_{1/2} = 0.01435$ ) E1  
 133.021 19 ( $t_{1/2} = 53.86$ ) E2  
 136.260 18 ( $t_{1/2} = 7.2723$ ) M1+E2:  $\delta = +0.413$   
 136.864 ( $t_{1/2} = 1.0723$ ) M1  
 345.936 ( $t_{1/2} = 18.7812$ ) E2  
 475.999 ( $t_{1/2} = 0.8737$ ) M2+E3:  $\delta = 0.51$   
 482.189 ( $t_{1/2} = 100.0014$ ) M1+E2:  $\delta = 4.764$   
 615.1711 ( $t_{1/2} = 0.29022$ ) M3(+E4)  
 618.668 ( $t_{1/2} = 0.031115$ ) (E2)

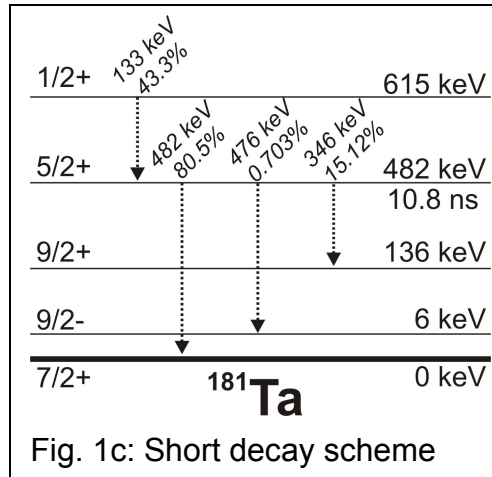
Fig. 1b: Transition intensities  $I_\gamma$  for the  $\beta^-$  decay of  $^{181}\text{Hf}$  (From: R. Firestone, *Table of Isotopes*, 8th edition, 1996).

## The principle of measuring half-lives

If we are taking only the part of our interest, with the strongest transitions of the decay scheme (Fig. 1c), we find the following:

The  $5/2^+$  state at 482 keV in  $^{181}\text{Ta}$ , whose half-life we want to measure, is being populated in most cases from the  $1/2^+$  state at 615 keV with a  $\gamma$ -ray of 133 keV. The de-population can occur in three ways by  $\gamma$ -rays with energies of 482 keV, 476 keV, or 346 keV. (Note the difference of the population of the  $1/2^+$  state by  $\beta^-$  decay (93%) and the depopulation via the 133 keV transition (43%). Where are the remaining 50%? How can an excited state being dis-excited, if not via  $\gamma$ -rays?)

For the determination of the half-life of the  $5/2^+$  state we measure the time intervals between the population of the 482 keV state by observing the 133 keV  $\gamma$ -rays and the time-delayed depopulation of the state by emission of the 482 keV  $\gamma$ -ray (we could also use the 476 keV or 346 keV lines, but they are weaker). These time intervals follow a special distribution (which one?), from which we can clearly determine the half-life of the excited state.



## Production of the $^{181}\text{Hf}$ sample in the FRM II

The  $^{181}\text{Hf}$  sample was produced by a 10min neutron irradiation of 23 mg of natural Hafnium metal ( $H_{180\text{Hf}} = 35.08\%$ ) in the research reactor FRM-II (Forschungsreaktor München II, <http://www.frm2.tum.de/>) on the campus.

The equation for the activation is

$$N_{181\text{Hf}}(0) = N_{\text{Hf}} \cdot H_{180\text{Hf}} \cdot \Phi \cdot t_{\text{act}} \cdot \sigma \cdot (1 - \exp(-\lambda \cdot t_{\text{act}})) \cdot 1/(\lambda \cdot t_{\text{act}})$$

The equation for the decay is

$$N_{181\text{Hf}}(t_{\text{wait}}) = N_{181\text{Hf}}(0) \cdot \exp(-\lambda \cdot t_{\text{wait}})$$

The first equation describes the production of  $^{181}\text{Hf}$  atoms by irradiation with a constant neutron flux  $\Phi$  [ $\text{n}/(\text{s} \cdot \text{cm}^2)$ ] for the time  $t_{\text{act}}$ .

The reaction cross section  $\sigma$  is for thermal neutrons with an energy of 25 meV  $\approx 13$  barn ( $1 \text{ barn} = 10^{-24} \text{ cm}^2$ ).

$H_{180\text{Hf}}$  is the abundance (in %) of  $^{180}\text{Hf}$  in nature (see Nuclide Table at [www.webelements.com](http://www.webelements.com)).

The term  $(1 - \exp(-\lambda \cdot t_{\text{act}})) \cdot 1/(\lambda \cdot t_{\text{act}})$  at the end of the first equation is for the fraction of  $^{181}\text{Hf}$  atoms which are already decayed during the irradiation time due to their half-life of  $t_{1/2} = 42.39$  d and are thus no longer available for the measurement. For long half-lives this term should be  $\approx 1$ , for short half-lives it is  $< 1$ . The decay constant  $\lambda$  can be calculated from  $\lambda = \ln(2) / t_{1/2}$ .

The second line considers the decay of  $^{181}\text{Hf}$  during the waiting time  $t_{\text{wait}}$ . For the transformation into the activity  $A$  (unit Bequerel; 1 Bq = 1 decay per second) the number of  $^{181}\text{Hf}$  atoms  $N_{181\text{Hf}}$  has to be multiplied with the decay constant:  $A = N \cdot \lambda$ .

## The detectors

The detectors are solid-state scintillation detectors, consisting of mono-crystals of inorganic elements. In our case we use sodium iodide doped with traces of thallium (NaI(Tl)).

Those scintillation detectors are used for the time-correlated detection of a single photon of ionizing radiation. They consist of two components: the scintillator converts the deposited energy into visible light. The photomultiplier (PM) then converts the light into an electronically detectable signal. The intensity of the light emitted from the scintillator is (almost) proportional to the energy of the incoming radiation.

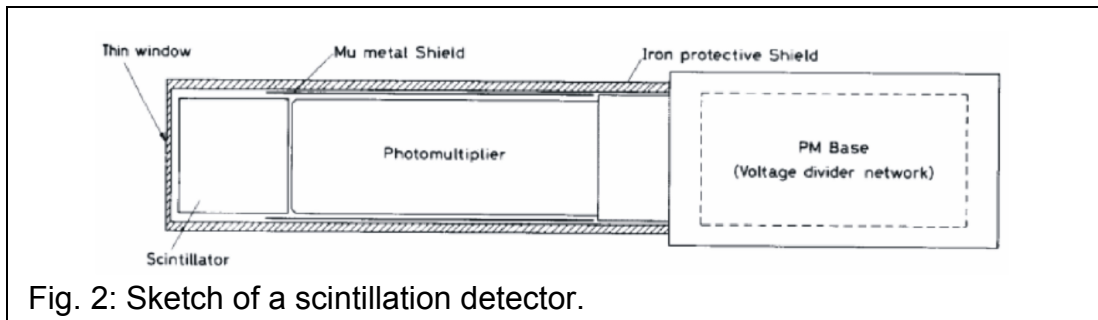


Fig. 2: Sketch of a scintillation detector.

The radioactive radiation, e.g. a  $\gamma$ -ray, excites electrons of the scintillator from the valence band into the conduction band and creates electron-hole pairs. These pairs migrate loss-free through the crystal until they reach a contamination. In the case of NaI(Tl) scintillation detectors this is a thallium atom. The electron-hole pair “decays”, the electron falls back into the valence band. The released energy is emitted as (optical) photon, and the light quanta are counted with the photomultiplier.

The gap between valence and conduction band is approx. 6-8 eV for alkali-iodide crystals. The entering photons have energies between 100 keV and some MeV, which is enough for the excitation of thousands electrons, or new photons. For being able to use these “new”  $\gamma$ -quanta they should have energies in the range of eV (in the order of the band gap) so they can be re-absorbed. Therefore the mono-crystal is doped with activator centers. The doping of the NaI with Tl deforms the conduction band locally and creates new energy levels in the band structure of the crystal, so-called “activator

bands”. They exhibit exactly the desired energy levels between valence and conduction band. Without activator band the electron would fall back into the valence band, and the emitted photon would have the same energy as the incoming photon. The scintillator would be impenetrable for the emitted light.

## Experimental setup

The experimental setup for measuring the time intervals is a so-called fast-slow coincidence. The full wiring diagram is shown in Fig. 3.

The inner circuit (“fast”) is for measuring the time intervals between the response of detector D1 and detector D2. By mean of the outer circuit (“slow”), from the multitude of measured time intervals those are chosen for which also the energy conditions are fulfilled. That means that in the start detector D1 a photon with the start energy (133 keV) and in the stop detector D2 a photon with the stop energy (482 keV) are registered. Only these events are further analyzed.

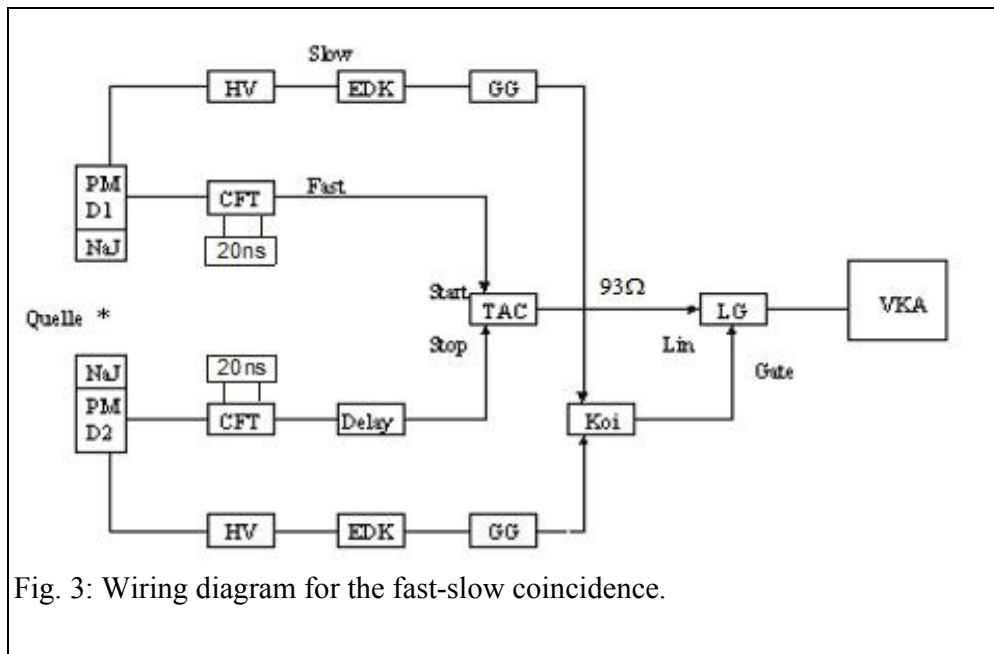


Fig. 3: Wiring diagram for the fast-slow coincidence.

Practically this works as follows: from the anodes of the photomultipliers (PM) the fast signals are taken. These signals are no longer proportional to the energy because of the high amplification factor, but due to their steep slope they are well suited for the control of the fast coincidence or – as in our case – of a time-to-amplitude converter (TAC). In constant fraction triggers (CFT) the fast signals are transformed into norm signals. (Note that the CFTs need an “inner delay” of 20 ns for being able to convert signals.) These are used as start and stop signal for the TAC. The TAC gives the time interval between the start and the stop signal as voltage amplitude to the output. The correct output to be used on the TAC is the 93Ω impedance.

For the slow or energy circuit the energy-proportional, slow signals from a lower dynode of the photomultiplier are used, because they are not yet saturated by the amplification of space charge. These signals are amplified and formed in the following main amplifiers (HV, Hauptverstärker!). (Take care: the label “HV” on the detectors is for the high voltage! Do not touch this!)

With the single channel discriminators (EDK, Einkanal-Diskriminatoren) the energies are selected which populate or depopulate the investigated level. If the  $\gamma$ -energy is within the selected window a logical signal is sent to the output. Because both single-channel signals have to overlap in the coincidence module (Koi), the time shifts of both signals due to runtime and electronics effects have to be considered and adjusted with gate generators (GG). The coincidence module gives only an output signal if both signals of the EDK are “synchronous”. Synchronous means relative to the time scale of the slow circuit, where delays of some ns are not measurable. The combination of slow and fast circuit occurs with help of the coincidence module (Koi) in the linear gate stretcher (LG). The time signal from the TAC passes the LG only if there is a positive signal from the Koi at the same time. Only time intervals which come from the start and the stop transition of the nucleus are sent to the multi-channel analyzer (VKA, Vielkanalanalysator).

## Performance of the measurements

### Part 1: Energy and efficiency calibration, measurement of the source activity, and time calibration

(a) We start with the energy spectrum.

Therefore we connect one amplifier (HV) output (e.g. from D2) directly with the VKA. For the energy and efficiency calibration two calibration sources ( $^{22}\text{Na}$ ,  $^{57}\text{Co}$ ) are available, whose energies and activities are well-known (see Fig. 4 and the certificate in the lab room).

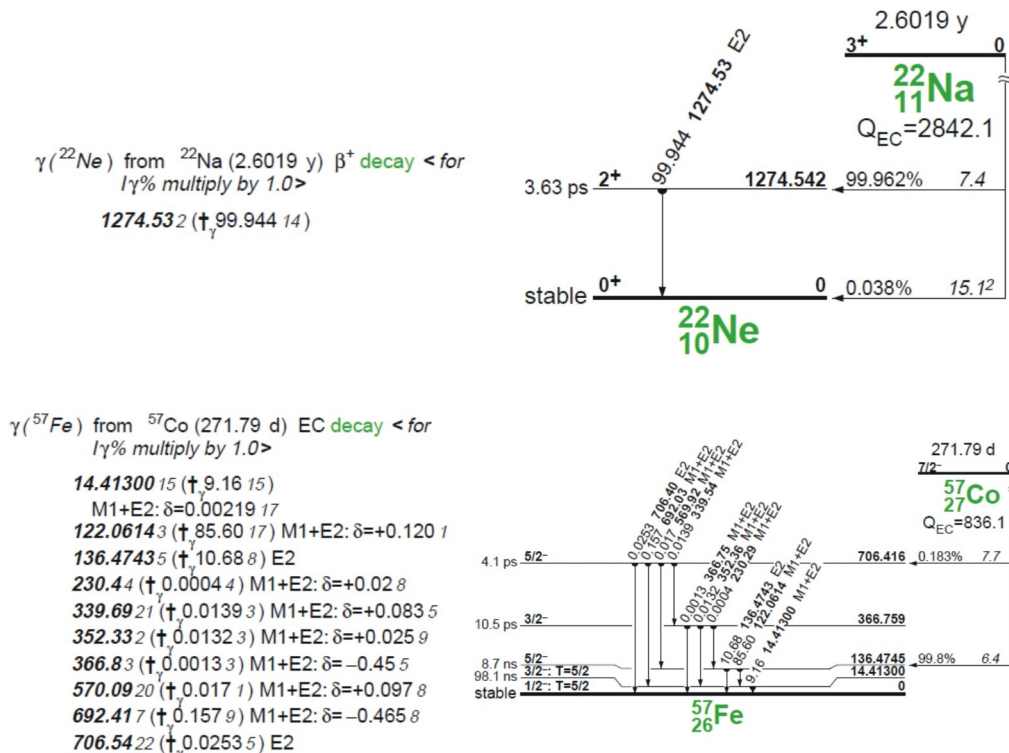


Fig. 4: Decay schemes of  $^{22}\text{Na}$  and  $^{57}\text{Co}$  (From: R. Firestone, *Table of Isotopes*, 8th edition, 1996).

We combine the efficiency and energy calibration. Record the spectra of the  $^{22}\text{Na}$  and  $^{57}\text{Co}$  for some minutes (take care of the “true time” and “life time” !) and determine the area under the strongest peaks. (You will see two peaks in the  $^{22}\text{Na}$  spectrum – why? Where does the second line come from?)

Then do the energy calibration at the computer with the strongest transitions of  $^{57}\text{Co}$  and  $^{22}\text{Na}$ . (The assistant will help you with this.)

The efficiency  $\varepsilon$  of the setup is strongly dependent on the distance between source and detector and the geometry. Therefore the measured count rate (counts/s)  $C$  is compared with the real activity  $A$  (in Bq) considering the transition probability  $I_\gamma$  (Fig. 1b):

$$\varepsilon = C / (A * I_\gamma)$$

Determine the efficiency for the three strongest transitions of both sources and plot it in a double-logarithmic graph:

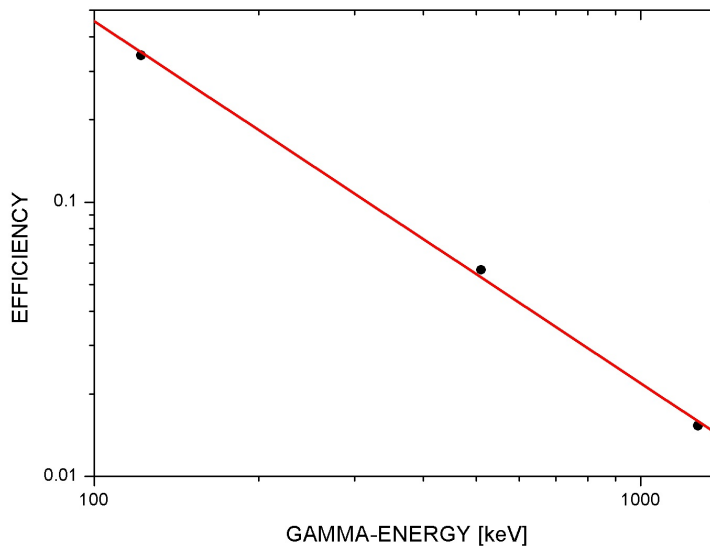


Fig. 5: Double-logarithmic plot of the efficiency.

The efficiencies for the 482 keV transition of  $^{181}\text{Hf}$  can then be determined by interpolation.

(b) Record the spectrum of the  $^{181}\text{Hf}$  sample. The activity  $A$  of the sample can be determined from the area under the 482 keV peak by transforming the equation for the efficiency above.

Calculate also the activity of the sample at the day of the irradiation (ask the assistant for the respective date) with the decay correction factor  $\exp(-\lambda * t_{\text{wait}})$ .

(c) For the determination of the time spectrum from which the half-life of the 482 keV state will be determined we need a time calibration (see Fig. 6).

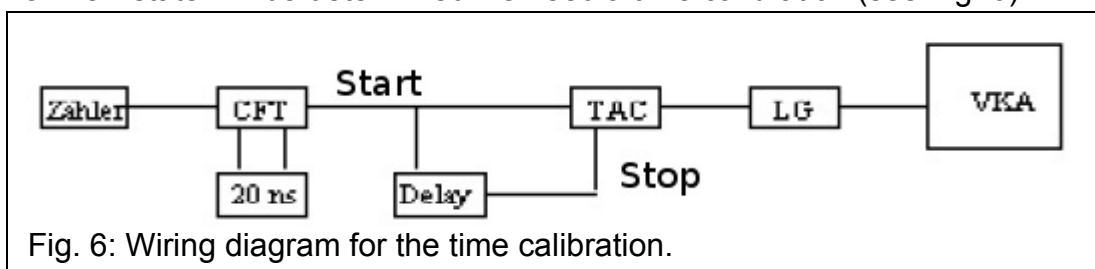


Fig. 6: Wiring diagram for the time calibration.

Therefore the output signals of one CFT are used. The same signal is connected once with the start of the TAC and once via several calibrated delays with the stop of the TAC. This produces for each delay a one-line spectrum which corresponds to the pulse height at the exit of the TAC. The LG has to be set to “NORMAL”. Do not forget to use 93Ω output on the TAC. Use the  $^{22}\text{Na}$  source and write down the delay and the respective channel for 20, 40, 60, and 80 ns. Plot the four data points linearly and determine the fit parameters. Why is the data point at 60 ns slightly below the fit curve?

Part 2: Time resolution of the TAC, real and random coincidences, prompt and delayed coincidences, measurement of the speed of light

(a) For the measurement of the prompt coincidence see the wiring diagram in Fig. 7. Both annihilation photons from the  $^{22}\text{Na}$  source are detected simultaneously in both detectors (why?). Despite the synchronism when the source is positioned exactly in the middle between both detector you will not see an one-line spectrum, but a time distribution with finite width, the so-called “prompt distribution”. The time resolution of the setup (the measure is the “full width at half maximum”, FWHM, of the prompt distribution) and the decline on both sides of the peak depend on the scintillators, the photomultipliers, the selected energy in the EDKs, the CFTs, and the TAC.

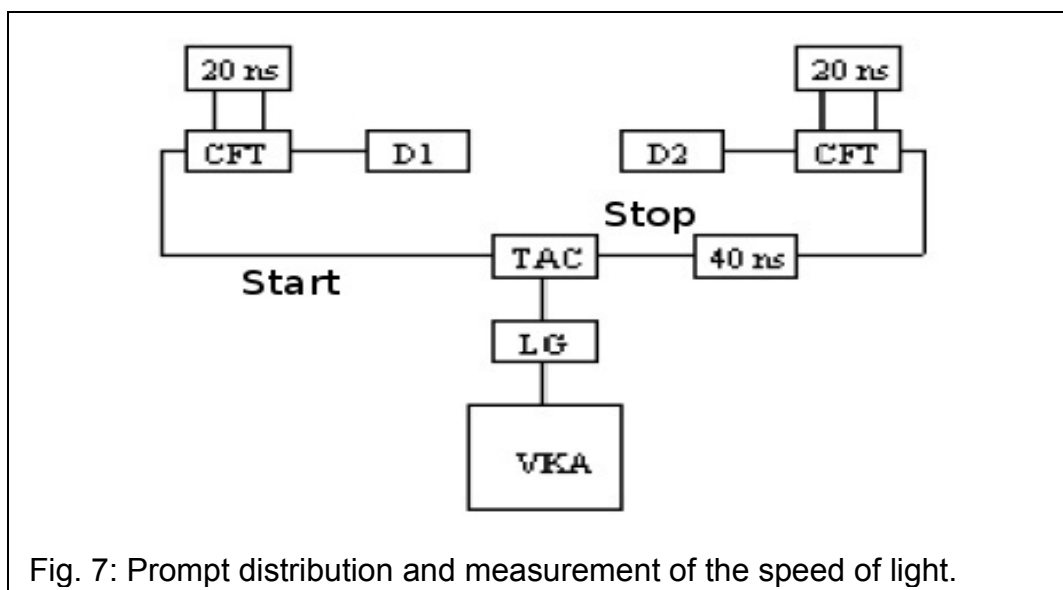


Fig. 7: Prompt distribution and measurement of the speed of light.

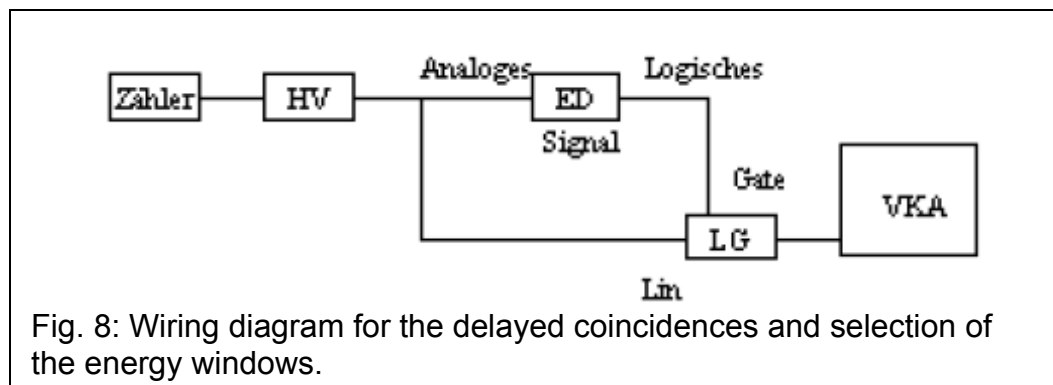
Half-lives of excited nuclear states can be measured directly by delayed coincidences if the FWHM of the prompt time distribution is smaller than the half-life which has to be determined. (is this the case for your setup? Estimate the FWHM in ns).

The count rate of the TAC consists of real coincidences, which come from the decay of the investigated nucleus, and random coincidences, which originate from different nuclei. The latter are thus not time-correlated.

(b) With the same setup we can also determine the speed of light. Therefore the  $^{22}\text{Na}$  source is being positioned asymmetrically between the two detectors

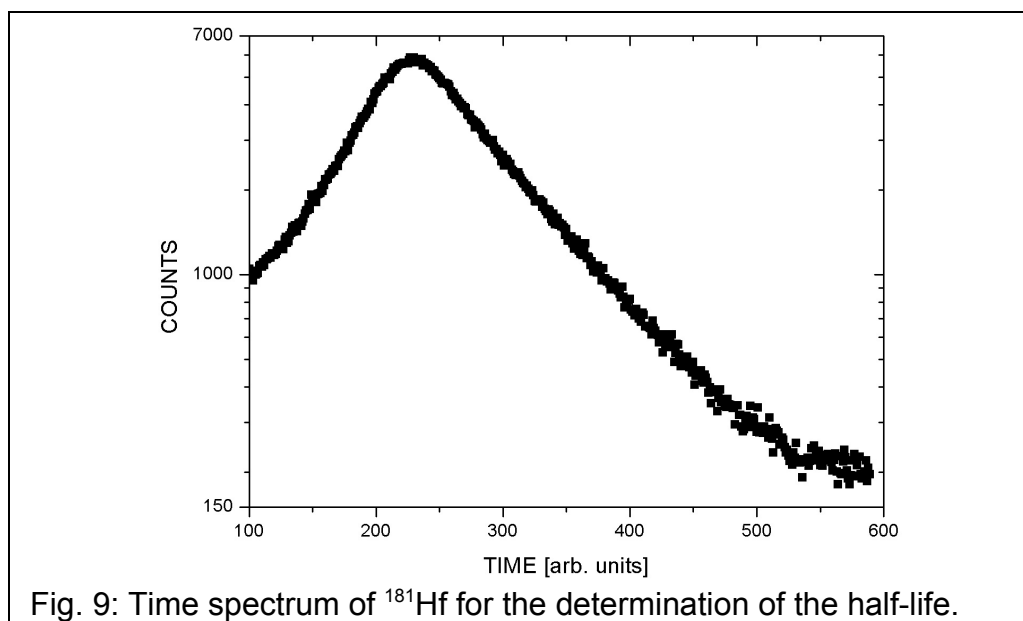
(distance 30-40 cm). Start the measurement and let it run for ~1h. (What changes in the prompt spectrum? Calculate the time of flight for the  $\gamma$ -ray for your chosen distance.)

(c) For determination of the delayed coincidence connect the HV output of one detector with the analog (linear) input of the EDK, and the output of the EDK with the “gate” input of the LG (Fig. 8). Set the LG back to “GATED”. The second output signal from the HV is being connected with the “linear” input of the LG (Note: There are two ways for the correct wiring between the HV and the EDK and the LG.).



Move both detectors together again and place the  $^{181}\text{Hf}$  source between them. You will see the energy spectrum on the computer screen. If the windows of the EDK are open, you see the full spectrum. By turning the buttons for the upper and lower threshold on the EDK you can select the respective peak. Start with D1 and select the start peak (133 keV), then repeat the procedure with D2 and the stop peak (482 keV).

(d) The final measurement of the half-life of the  $5/2^+$  state in  $^{181}\text{Ta}$  is done with the full wiring diagram in Fig. 3. The used delay is 40 ns (How does the spectrum change if you choose another delay?). An example for the spectrum is shown in Fig. 9. Which distribution(s) do you see? How can one determine the half-life from this?





## Analysis of the measurements

Please do not forget to write down the necessary information in the single steps (life time, real time, area of peaks, channels...)! There is no need for an extensive error calculation with Gaussian error propagation. Estimate the errors where necessary.

- Make an efficiency plot (Fig. 5) and determine the efficiency for the 482 keV line.
- Determine the activity of the  $^{181}\text{Hf}$  source at the day of the experiment and the day of irradiation.
- Plot and fit the time calibration spectrum.
- Determine the exact FWHM (in ns) of the prompt coincidence spectrum with the fit function from the time calibration.
- Calculate the speed of light and compare it with the “real value”. How long does the light need for the distance between the two detectors?
- Determine the half-life of the  $5/2^+$  state (Fig. 9) with a fit. The assistant will send you your measured spectrum.

## Additional questions

a) Compare your measured half-life with the single-particle estimate from Victor Weisskopf (1908 – 2002). Explain shortly what the Weisskopf estimate is (~4 sentences). The formulars are splitted into different multipolarities (see the brackets in Fig. 1a):

$$t_{1/2}^w (\text{M1}) = 2.24 \cdot 10^{-14} E_\gamma^{-3} \text{ [s]}$$

$$t_{1/2}^w (\text{E2}) = 9.37 \cdot 10^{-9} E_\gamma^{-5} A^{-3/4} \text{ [s]}$$

$A$  is the mass number and  $E_\gamma$  the  $\gamma$ -energy in MeV. Calculate the Weisskopf estimate for the 133 keV and the 482 keV transition (add the two values when two multipolarities are given). What is the “reference value” for the 133 keV transition? How good does the Weisskopf estimate describe the “real” values? How differs the reality (nuclear excitation) from the assumptions Weisskopf has made?

b) When does a  $\gamma$ - $\gamma$  angular correlation occur? How can the angular correlation be disturbed and falsify the half-life measurement with this method?

c) Natural radioactivity: which radioactive isotopes are responsible for the main body activity? How much activity is in a standard body of 70 kg? (And for those who are interested: Why is the activity in a female body of the same weight different from a male body?)

For this please download and read the following script:

Martin Volkmer: „Radioaktivität und Strahlenschutz“:

[http://www.kernenergie.de/kernenergie/documentpool/Service/013radioaktivitaet\\_strahlenschutz2007.pdf](http://www.kernenergie.de/kernenergie/documentpool/Service/013radioaktivitaet_strahlenschutz2007.pdf)

Chapter 7.7 describes the natural activity of a standard human. Please re-calculate the values of the two most important radioactive isotopes in Table 7-8 on your own. Make a table like the following and calculate the body activity:

| Element | Radio-isotope | % per body | kg per 70 kg | Radioisotope per element | g Radiois. per 70 kg | Atoms | Half-life [y] | Decay constant $\lambda$ [1/s] | Activity [Bq] |
|---------|---------------|------------|--------------|--------------------------|----------------------|-------|---------------|--------------------------------|---------------|
|         |               |            |              | 1.00E-12                 |                      |       | 5730          |                                |               |
|         |               |            |              | 0.0117%                  |                      |       | 1.28E+09      |                                |               |

For the % of the element per body see

“Grundstoffe im menschlichen Körper“

[http://www.rodiehr.de/d\\_03\\_grundstoffe\\_im\\_koerper.htm](http://www.rodiehr.de/d_03_grundstoffe_im_koerper.htm)

f) Short critique: Did something displease you? What can the assistant do to improve this experiment? (Note: New computer is coming soon!).

## Safety Instructions

The experiment does not have concrete hazards. However, you must be aware that you operate high voltage device and radioactive sources. Furthermore, general hygienic rules for experimental areas should be considered.

In details:

1. the photomultipliers work at voltage as high as 2000 V. The current limit of the high voltage generators is lower than 1 mA, safe for humans, however there is no reason to touch the high voltage cables and the high voltage generator. If you feel something wrong on it, call your tutor, do not touch high-voltage related parts. The tutor will show you exactly where these parts are located.
2. You operate three radioactive source:  $^{22}\text{Na}$ ,  $^{57}\text{Co}$ ,  $^{181}\text{Hf}$ . All of these source are sealed, therefore there is no hazard of contamination. All of these sources are gamma emitter with energy between 100 keV and 1300 keV. Gamma of such energy has low probability of interacting with human tissue. Furthermore, the activity of these source is always lower than 300 kBq. Such radioactivity amount does not consist a considerable hazard and even not require the use of a dosimeter. However, the rule of “As Low As Reasonable Achievable (ALARA)” dose has always to be followed. Therefore you handle the source only for the time you need to position it (but you do not have to rush!), you do not stay closer than 1 m distance from the source during the measurements, and you put back the source in the shielded container after the measurements. You can perform the operations above by yourself, but in case of any doubt or complain please call your tutor.

3. To shield the sources during the measurements, a lead sheet of about 1 kg is available. Large amount of lead powder is dangerous for the lungs, but only 1 kg of lead can not ever be considered an hazard. However, it is worth to clean your hands after managing lead, and avoid contact with nose and mouth.

As a general rule, considering also other sources of dust and powder, eating and drinking in the experimental area is not allowed, and you are requested to wash your hands when you exit the area, especially in case of meal break.

## Literature

- Webelements: <http://www.webelements.com/>
- Lederer et al.: *Table of Isotopes*; new version: R. Firestone, *Table of Isotopes*, 8th edition, 1996
- Decay radiation search:  
[http://www.nndc.bnl.gov/nudat2/indx\\_dec.jsp](http://www.nndc.bnl.gov/nudat2/indx_dec.jsp)
- Knoll: *Radiation Detection and Measurement* (3<sup>rd</sup> edition, John Wiley & Sons, 1999)
- Mayer-Kuckuck: *Kernphysik*
- Musiol, Ranft, Reif, Seeliger: *Kern- und Elementarteilchenphysik*