Tabelle mit Nabla-Operator in Zylinder und Kugelkoordinaten

Operation	Kartesische Koordinaten (x,y,z)	Zylinderkoordinaten (ρ,φ,z)	Kugelkoordinaten (r,θ,φ)
Definition der Koordinaten		$x = \rho\cos\phi$ $y = \rho\sin\phi$ $z = z$	$x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$
		$\rho = \sqrt{x^2+y^2}$ $\phi = \operatorname{atan2}(y,x)$ $z = z$	$r = \sqrt{x^2+y^2+z^2}$ $\theta = \arccos(z/r)$ $\phi = \operatorname{atan2}(y,x)$
$\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\boldsymbol{z}}$	$A_r\hat{\boldsymbol{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	$\frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$
$\nabla\cdot\mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla\times\mathbf{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}}$ + $\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}}$ + $\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\boldsymbol{\rho}}$ + $\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\phi}}$ + $\frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}}$ + $\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}}$ + $\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}$
$\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
$\Delta\mathbf{A} = \nabla^2\mathbf{A}$	$\Delta A_x\hat{\mathbf{x}} + \Delta A_y\hat{\mathbf{y}} + \Delta A_z\hat{\mathbf{z}}$	$(\Delta A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{\rho}}$ + $(\Delta A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi})\hat{\boldsymbol{\phi}}$ + $(\Delta A_z)\hat{\boldsymbol{z}}$	$(\Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta\cos\theta}{r^2\sin\theta} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{r}}$ + $(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{\theta}}$ + $(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin^2\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi})\hat{\boldsymbol{\phi}}$
infinitesimale Verschiebung	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$

infinitesimales Flächenelement	$d\mathbf{A} = dydz\hat{\mathbf{x}} +$ $dxdz\hat{\mathbf{y}} +$ $dxdy\hat{\mathbf{z}}$	$d\mathbf{A} = \rho d\phi dz\hat{\boldsymbol{\rho}} +$ $d\rho dz\hat{\boldsymbol{\phi}} +$ $\rho d\rho d\phi\hat{\mathbf{z}}$	$d\mathbf{A} = r^2\sin\theta d\theta d\phi\hat{\mathbf{r}} +$ $r\sin\theta dr d\phi\hat{\boldsymbol{\theta}} +$ $rdrd\theta\hat{\boldsymbol{\phi}}$
infinitesimales Volumenelement	$dV = dxdydz$	$dV = \rho d\rho d\phi dz$	$dV = r^2\sin\theta dr d\theta d\phi$

Nichttriviale Rechenregeln:

1. $\operatorname{div}\operatorname{grad} f = \nabla\cdot(\nabla f) = \nabla^2 f = \Delta f$ (Laplace-Operator)
2. $\operatorname{rot}\operatorname{grad} f = \nabla\times(\nabla f) = 0$
3. $\operatorname{div}\operatorname{rot}\mathbf{A} = \nabla\cdot(\nabla\times\mathbf{A}) = 0$
4. $\operatorname{rot}\operatorname{rot}\mathbf{A} = \nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \Delta\mathbf{A}$
5. $\Delta(fg) = f\Delta g + 2\nabla f\cdot\nabla g + g\Delta f$
6. $\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A})$,
   woraus mit $\mathbf{A} = \mathbf{B} = \mathbf{v}$ unmittelbar die für die Strömungslehre wichtige Weber-Transformation folgt:
   $(\mathbf{v}\cdot\nabla)\mathbf{v} = \nabla\frac{\mathbf{v}^2}{2} - \mathbf{v}\times(\nabla\times\mathbf{v})$
7. $\mathbf{A}\times(\nabla\times\mathbf{C}) = \nabla_{\mathbf{C}}(\mathbf{A}\cdot\mathbf{C}) - (\mathbf{A}\cdot\nabla)\mathbf{C} = (\nabla\mathbf{C})\cdot\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{C}$
8. $\nabla\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\nabla\cdot\mathbf{C}) - \mathbf{C}(\nabla\cdot\mathbf{B}) + (\mathbf{C}\cdot\nabla)\mathbf{B} - (\mathbf{B}\cdot\nabla)\mathbf{C}$