

Tabelle mit Nabla-Operator in Zylinder und Kugelkoordinaten

Operation	Kartesische Koordinaten (x,y,z)	Zylinderkoordinaten (ρ,φ,z)	Kugelkoordinaten (r,θ,φ)
Definition der Koordinaten		$\begin{cases} x = \rho \cos \phi \\ y = \rho \sin \phi \\ z = z \end{cases}$	$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$
		$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \phi = \operatorname{atan2}(y, x) \\ z = z \end{cases}$	$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arccos(z/r) \\ \phi = \operatorname{atan2}(y, x) \end{cases}$
A	$A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$	$A_\rho \hat{\rho} + A_\phi \hat{\phi} + A_z \hat{z}$	$A_r \hat{r} + A_\theta \hat{\theta} + A_\phi \hat{\phi}$
∇f	$\frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$	$\frac{\partial f}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$	$\frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \hat{\phi}$
$\nabla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho} \frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$
$\nabla \times \mathbf{A}$	$\begin{pmatrix} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{pmatrix} \hat{x} + \dots$	$\begin{pmatrix} \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} - \frac{\partial A_z}{\partial z} \\ \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \\ \frac{1}{\rho} \left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right) \end{pmatrix} \hat{\rho} + \dots$	$\frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta}(A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right) \hat{r} + \dots$
$\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$
$\Delta \mathbf{A} = \nabla^2 \mathbf{A}$	$\Delta A_x \hat{x} + \Delta A_y \hat{y} + \Delta A_z \hat{z}$	$\begin{pmatrix} \Delta A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2} \frac{\partial A_\phi}{\partial \phi} \\ \Delta A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2} \frac{\partial A_\rho}{\partial \phi} \\ \Delta A_z \end{pmatrix} \hat{\rho} + \dots$	$\begin{pmatrix} \Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta \cos \theta}{r^2 \sin \theta} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \\ \Delta A_\theta - \frac{A_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi} \\ \Delta A_\phi - \frac{A_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin^2 \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\theta}{\partial \phi} \end{pmatrix} \hat{r} + \dots$
infinitesimale Verschiebung	$d\mathbf{l} = dx \hat{x} + dy \hat{y} + dz \hat{z}$	$d\mathbf{l} = d\rho \hat{\rho} + \rho d\phi \hat{\phi} + dz \hat{z}$	$d\mathbf{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$

infinitesimales Flächenelement	$d\mathbf{A} = dydz \hat{x} + dx dz \hat{y} + dx dy \hat{z}$	$d\mathbf{A} = \rho d\phi dz \hat{\rho} + \rho dz d\phi \hat{\phi} + \rho d\rho d\phi \hat{z}$	$d\mathbf{A} = r^2 \sin \theta d\theta d\phi \hat{r} + r \sin \theta dr d\phi \hat{\theta} + r dr d\theta \hat{\phi}$
infinitesimales Volumenelement	$dV = dx dy dz$	$dV = \rho d\rho d\phi dz$	$dV = r^2 \sin \theta dr d\theta d\phi$

Nichttriviale Rechenregeln:

- $\operatorname{div} \operatorname{grad} f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$ (Laplace-Operator)
- $\operatorname{rot} \operatorname{grad} f = \nabla \times (\nabla f) = 0$
- $\operatorname{div} \operatorname{rot} \mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
- $\operatorname{rot} \operatorname{rot} \mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \Delta \mathbf{A}$
- $\Delta(fg) = f \Delta g + 2 \nabla f \cdot \nabla g + g \Delta f$
- $\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$,
woraus mit $\mathbf{A} = \mathbf{B} = \mathbf{v}$ unmittelbar die für die Strömungslehre wichtige Weber-Transformation folgt:
 $(\mathbf{v} \cdot \nabla) \mathbf{v} = \nabla \frac{v^2}{2} - \mathbf{v} \times (\nabla \times \mathbf{v})$
- $\mathbf{A} \times (\nabla \times \mathbf{C}) = \nabla_{\mathbf{C}}(\mathbf{A} \cdot \mathbf{C}) - (\mathbf{A} \cdot \nabla) \mathbf{C} = (\nabla \mathbf{C}) \cdot \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{C}$
- $\nabla \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\nabla \cdot \mathbf{C}) - \mathbf{C}(\nabla \cdot \mathbf{B}) + (\mathbf{C} \cdot \nabla) \mathbf{B} - (\mathbf{B} \cdot \nabla) \mathbf{C}$

Von „http://de.wikipedia.org/wiki/Formelsammlung_Nabla-Operator“

Kategorien: Analysis | Formelsammlung | Liste (Mathematik)

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